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Title: *Regularity via duality*

Abstract

I will present a technique to prove some regularity results for the solutions of variational problems in duality (and for the corresponding Euler-Lagrange PDEs) that we discovered hidden in a work by Y. Brenier on fluid mechanics. When we have a minimization problem $\min\{A(u), u \in X\}$ with a dual problem $\max\{-B(\phi), \phi \in Y\}$, with optimizers u_0 and ϕ_0 we can "test" the primal-dual relation on translations $u_h := u_0(x+h)$ of the optimal u_0 , and this can give informations on $\|u_h - u_0\|$, i.e. Sobolev regularity for u_0 .

I will show some concrete examples, starting from the Poisson equation $\Delta u = f$, and then passing to more degenerate problems, of p-Laplace type, but also problems involving time, or transport costs.

the technique is not optimal at all, as it only gives very poor regularity results (typically of H^1 type), but has the advantage that it also works under very weak assumptions.